Basic Math

Question1

If $p^3 = q^4 = r^6 = t^7 = s^2$, then $\log_t(pqrs) = \ldots.$ MHT CET 2025 (26 Apr Shift 2)

Options:

A. $\frac{168}{5}$

B. 28

C. $\frac{31}{4}$

D. $\frac{35}{4}$

Answer: D

Solution:

Given the equation $p^3 = q^4 = r^6 = t^7 = s^2$, we are to find $\log_t(pqrs)$.

Let the common value be k, i.e.,
$p^3 = q^4 = r^6 = t^7 = s^2 = k$

From this, express each base in terms of k and exponents:

$$p = k^{\frac{1}{3}}, \quad q = k^{\frac{1}{4}}, \quad r = k^{\frac{1}{6}}, \quad t = k^{\frac{1}{7}}, \quad s = k^{\frac{1}{2}}$$

Calculate $pqrs$:

$$pqrs = k^{\frac{1}{3}} \times k^{\frac{1}{4}} \times k^{\frac{1}{6}} \times k^{\frac{1}{2}} = k^{\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{2}\right)}$$

Find sum of exponents:

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2} = \frac{4}{12} + \frac{3}{12} + \frac{2}{12} + \frac{6}{12} = \frac{15}{12} = \frac{5}{4}$$

So,

$$pqrs = k^{\frac{5}{4}}$$

We are to find $\log_t(pqrs)$.

Since $t = k^{\frac{1}{7}}$, then

$$\log_t(pqrs) = \frac{\log_k(k^{\frac{5}{4}})}{\log_k(k^{\frac{1}{7}})} = \frac{\frac{5}{4}}{\frac{1}{7}} = \frac{5}{4} \times 7 = \frac{35}{4}$$

Hence, the answer is $\frac{35}{4}$.

The correct choice is option D.

Question2

If $\sqrt{\log_3 x^{16}} + 9\log_{27}\sqrt[3]{\frac{3}{x}} = 5$then $x = \ldots$.. MHT CET 2025 (21 Apr Shift 2)

Options:

A. 81

B. $\frac{1}{405}$

C. 27

D. 405

Answer: A

Solution:

Given:

$$\sqrt{\log_3 x^{16}} + 9\log_{27}\sqrt{\frac{3}{x}} = 5$$

Rewrite terms:

$$4\sqrt{\log_3 x} + 9\left(\frac{1}{9} - \frac{1}{3}\log_{27} x\right) = 5$$

Simplify:

$$4\sqrt{\log_3 x} + 1 - 3\log_{27} x = 5 \implies 4\sqrt{\log_3 x} - 3\log_{27} x = 4$$

Since $\log_{27} x = \frac{\log_3 x}{3}$, put $y = \sqrt{\log_3 x}$ so $\log_3 x = y^2$.

Equation becomes:

$$4y - y^2 = 4 \implies y^2 - 4y + 4 = 0 \implies (y-2)^2 = 0 \implies y = 2$$

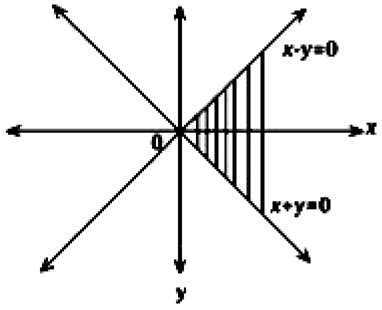
So,

$$\sqrt{\log_3 x} = 2 \implies \log_3 x = 4 \implies x = 81$$

Answer: $x = 81$

Question3

The shaded region in the following figure represents a solution set of



MHT CET 2025 (20 Apr Shift 1)

Options:

A. $x - y > 0, x + y > 0$

B. $x - y \leq 0, x + y > 0$

C. $x - y > 0, x + y \leq 0$

D. $x - \mathrm{y} < 0, x + \mathrm{y} < 0$

Answer: A

Solution:

We are given the graph where the shaded region is bounded by the lines:

- $x - y = 0 \quad \Rightarrow \quad x = y$
- $x + y = 0 \quad \Rightarrow \quad x = -y$

The shaded region lies:

- Above the line $x - y = 0$, i.e., $x - y > 0 \Rightarrow x > y$.
- Above the line $x + y = 0$, i.e., $x + y > 0 \Rightarrow x > -y$.

So, the solution set is:

$$x - y > 0, \quad x + y > 0$$

✅ Correct Option: A. $x - y > 0,\ x + y > 0$

Question4

If $x + \log_{15}(5 + 3^x) = x\log_{15}5 + \log_{15}24$, then $x = \ldots..$ MHT CET 2025 (19 Apr Shift 1)

Options:

A. 1

B. 5

C. 2

D. 8

Answer: A

Solution:

Given:

$$x + \log_{15}(5 + 3^2) = x\log_{15}5 + \log_{15}24$$

$$x + \log_{15}14 = x\log_{15}5 + \log_{15}24$$

$$x(1 - \log_{15}5) = \log_{15}\frac{24}{14} = \log_{15}\frac{12}{7}$$

$$1 - \log_{15}5 = \log_{15}3$$

$$x\log_{15}3 = \log_{15}\frac{12}{7}$$

$$x = \log_3 \frac{12}{7} \approx 1$$

✅ Answer: 1

Question5

The assets of a person are reduced in his business such that the rate of reduction is proportional to the square root of the existing assets. If the assets were initially ₹ $10,00,000$ and due to loss they reduce to ₹ 10,000 after 3 years, then the number of years required for the person to go bankrupt will be MHT CET 2024 (16 May Shift 1)

Options:

A. $\frac{10}{3}$

B. $\frac{10}{9}$

C. $\frac{20}{9}$

D. $\frac{20}{3}$

Answer: A

Solution:

Let x be the asset at time t .

$$\therefore \quad \frac{\mathrm{d}x}{\mathrm{dt}} \propto \sqrt{x}$$

$$\Rightarrow \frac{\mathrm{d}x}{\mathrm{dt}} = -\mathrm{k}\sqrt{x}, \text{ where } \mathrm{k} > 0$$ Integrating on both sides,

$$\Rightarrow \frac{\mathrm{d}x}{\sqrt{x}} = -\mathrm{kdt}$$

$$2\sqrt{x} = -\mathrm{kt} + \mathrm{c}$$

$$\text{When t} = 0, x = 10,00,000$$

we get $\therefore \quad 2\sqrt{1000000} = -\mathrm{k}(0) + \mathrm{c}$

$$\Rightarrow \quad \Rightarrow \mathrm{c} = 2(1000) = 2000$$

$$\therefore \quad 2\sqrt{x} = -\mathrm{kt} + 2000 \ldots (i)$$

$$\text{When t} = 3, x = 10,000$$

$$\therefore \quad 2\sqrt{10000} = -3\mathrm{k} + 2000$$

$$\Rightarrow 2(100) = -3\mathrm{k} + 2000$$

$$\Rightarrow 3\mathrm{k} = 1800$$

$$\Rightarrow \mathrm{k} = 600$$

$$\therefore \quad 2\sqrt{x} = -600\mathrm{t} + 2000 \ldots [From(i)]$$

Time to go bankrupt $= \text{TWhen t} = \mathrm{T}, x = 0$

$$\therefore \quad 0 = -600\ \mathrm{T} + 2000$$

$$\Rightarrow \mathrm{T} = \frac{2000}{600} = \frac{10}{3} \text{ years}$$

Question6

If $y = \frac{x^{\frac{2}{3}} - x^{\frac{-1}{3}}}{x^{\frac{2}{3}} + x^{\frac{-1}{3}}}$, $x \neq 0$, then $(x+1)^2 y_1 =$ MHT CET 2024 (15 May Shift 2)

Options:

A. 2

B. -2

C. $\frac{-1}{3}$

D. 3

Answer: A

Solution:

$$\begin{aligned} y &= \frac{x^{\frac{2}{3}} - x^{-\frac{1}{3}}}{x^{\frac{2}{3}} + x^{-\frac{1}{3}}} \\ &= \frac{x^{-\frac{1}{3}}(x-1)}{x^{-\frac{1}{3}}(x+1)} \end{aligned}$$

$$\therefore \quad y = \frac{x-1}{x+1}$$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(x+1)^2}$$

$$\Rightarrow (x+1)^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 2$$

Question7

The approximate value of $\log_{10} 1002$ is (Given $\log_{10} \mathrm{e} = 0.4343$)
MHT CET 2024 (04 May Shift 1)

Options:

A. 3.0117

B. 3.0009

C. 2.9999

D. 3.1119

Answer: B

Solution:

$$\text{Let } \mathrm{f}(x)=\log_e x$$
$$\therefore \quad \mathrm{f}'(x)=\frac{1}{x}$$
$$\text{Here, } a=1000,\ \mathrm{h}=2$$
$$\therefore \quad \mathrm{f}(\mathrm{a}+\mathrm{h}) \approx \mathrm{f}(\mathrm{a})+\mathrm{hf}'(\mathrm{a})$$
$$\approx f(1000)+2 f'(1000)$$
$$\approx \log_{\mathrm{e}} 10^3+2 \times \frac{1}{1000}$$
$$\approx 3(0.4343)+0.002$$
$$\approx 1.3029+0.002$$
$$\approx 1.3049$$
$$\therefore \quad \log_{\mathrm{e}} 1002 \approx 1.3049$$
$$\text{Now, } \log_{10} 1002 \approx \frac{\log_{\mathrm{e}} 1002}{\log_{\mathrm{e}} 10}$$
$$\approx \frac{1.3049}{0.4343} \approx 3.0009$$

Question8

If $y=[(x+1)(2x+1)(3x+1)\ldots\ldots\ldots\ldots(\mathrm{n}x+1)]^4$ **then** $\frac{\mathrm{d}y}{\mathrm{d}x}$ **at** $x=0$ **is MHT CET 2024 (03 May Shift 2)**

Options:

A. $\frac{\mathrm{n}(\mathrm{n}+1)}{2}$

B. $4\mathrm{n}(\mathrm{n}+1)$

C. $\left(\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right)^2$

D. $2\mathrm{n}(\mathrm{n}+1)$

Answer: D

Solution:

$$y=[(x+1)(2x+1)(3x+1)\ldots(nx+1)]^4$$
$$\Rightarrow \log y=4[\log(x+1)(2x+1)(3x+1)\ldots(nx+1)]$$
$$\Rightarrow \quad \log y=4[\log(x+1)+\log(2x+1)$$
$$+\log(3x+1)+\ldots+\log(nx+1)]$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=4\left[\frac{1}{x+1}+\frac{2}{2x+1}+\frac{3}{3x+1}+\ldots+\frac{\mathrm{n}}{\mathrm{n}x+1}\right]$$
$$\Rightarrow \frac{1}{1}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=0}=4(1+2+3+\ldots\mathrm{n})$$
$$\Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=0}=4\left(\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right)=2\mathrm{n}(\mathrm{n}+1)$$

Question9

If a body cools from $80^\circ C$ to $50^\circ C$ in the room temperature of $25^\circ C$ in 30 minutes, then the temperature of the body after 1 hour is MHT CET 2023 (14 May Shift 2)

Options:

A. $31.36^\circ C$

B. $32.25^\circ C$

C. $36.36^\circ C$

D. $33.25^\circ C$

Answer: C

Solution:

Let θ be the temperature of the body at any time t.

$$\therefore \quad \frac{d\theta}{dt} \propto (\theta - 25)$$

$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - 25), k > 0$$

Integrating on both sides, we get

$$\log|\theta - 25| = -kt + c$$

$$\text{When } t = 0, \theta = 80^\circ$$

$$\therefore \quad \log 55 = 0 + c$$

$$\Rightarrow \quad c = \log 55 \log|\theta - 25| = -kt + \log 55$$

$$\therefore \quad \text{When } t = 30, \theta = 50^\circ$$

$$\therefore \quad \log 25 = -30k + \log 55$$

$$\Rightarrow k = -\frac{1}{30}\log\frac{5}{11}$$

$$\therefore \quad \log|\theta - 25| = \frac{t}{30}\log\frac{5}{11} + \log 55$$

When t $= 1$ hour $= 60$ minutes, ... [From (i)]

$$\log|\theta - 25| = \frac{60}{30}\log\frac{5}{11} + \log 55$$

$$\Rightarrow \log\left(\frac{\theta - 25}{55}\right) = 2\log\left(\frac{5}{11}\right)$$

$$\Rightarrow \frac{\theta - 25}{55} = \left(\frac{5}{11}\right)^2$$

$$\Rightarrow \frac{\theta - 25}{55} = \frac{25}{121}$$

$$\Rightarrow \theta = 25 + \frac{125}{11} = 25 + 11.36 = 36.36^\circ\text{C}$$

Question10

The money invested in a company is compounded continuously. If ₹200 invested today becomes ₹400 in 6 years, then at the end of 33 years it will become ₹ MHT CET 2023 (14 May Shift 1)

Options:

A. $1600\sqrt{2}$

B. $3200\sqrt{2}$

C. $12800\sqrt{2}$

D. $6400\sqrt{2}$

Answer: D

Solution:

Here, Amount $(A) = ₹400$Principal $(P) = ₹200$, $N = 6$ years

$$A - P\left(1+\frac{R}{100}\right)^{N}$$

$$\Rightarrow 400 = 200\left(1+\frac{R}{100}\right)^{6}$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^{6} = 2$$

$$\Rightarrow 1+\frac{R}{100} = 2^{\frac{1}{6}}$$

$$A = P\left(1+\frac{R}{100}\right)^{N}$$

$$= 200\left(1+\frac{R}{100}\right)^{33}$$

$$= 200\left(2^{\frac{1}{6}}\right)^{33}$$

$$= 200\left(2^{5}\cdot 2^{\frac{1}{2}}\right)$$

$$= 200(32\sqrt{2})$$

$$= 6400\sqrt{2}$$

Question11

The decay rate of radio active material at any time t is proportional to its mass at that time. The mass is 27 grams when $t = 0$. After three hours it was found that 8 grams are left. Then the substance left after one more hour is MHT CET 2023 (12 May Shift 1)

Options:

A. $\frac{27}{8}$ grams

B. $\frac{81}{4}$ grams

C. $\frac{16}{3}$ grams

D. $\frac{16}{9}$ grams

Answer: C

Solution:

Let ' x ' be the mass of the material at time ' t '.
$\therefore \quad \frac{\mathrm{d}x}{\mathrm{dt}} = -\mathrm{k}x$, ($-$ve sign indicates decay.)

$$\therefore \quad \int \frac{\mathrm{d}x}{x} = -\mathrm{k} \int \mathrm{dt}$$

$\therefore \quad \log |x| = -\mathrm{kt} + \mathrm{c}$When $\mathrm{t} = 0, x = 27. \therefore \quad \mathrm{c} = \log 27$
$\therefore \quad \log |x| = -\mathrm{kt} + \log 27$When $\mathrm{t} = 3, x = 8$

$$\therefore \quad \mathrm{k} = \log\left(\frac{3}{2}\right)$$

When $\mathrm{t} = 4$, we get

$$\log |x| = -4 \log\left(\frac{3}{2}\right) + \log 27$$
$$\therefore \quad \log |x| = \log\left(\frac{16}{3}\right)$$
$$\therefore \quad x = \frac{16}{3} \text{ grams}$$

Question12

If $\log_2 x + \log_4 x + \log_y x + \log_{16} x - \frac{25}{36}$ and $x = 2^k$, then k is MHT CET 2023 (09 May Shift 2)

Options:

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{8}$

Answer: C

Solution:

$$\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$$
$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 8} + \frac{\log x}{\log 16} = \frac{25}{36}$$
$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{3\log 2} + \frac{\log x}{4\log 2} = \frac{25}{36}$$
$$\frac{\log x}{\log 2}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = \frac{25}{36}$$

But $x = 2^{\mathrm{k}}$ ...

$$\frac{\log x}{\log 2}\left[\frac{25}{12}\right] = \frac{25}{36}$$
$$\frac{25}{12}\log_2 x = \frac{25}{36}$$
$$\log_2 x = \frac{1}{3}$$
$$\therefore \quad x = 2^{\frac{1}{3}}$$

[Given]$\therefore \quad \mathrm{k} = \frac{1}{3}$

Question13

If $|3x-2| \leq \frac{1}{2}$ then $x \in$ MHT CET 2020 (12 Oct Shift 1)

Options:

A. $\left[\frac{1}{2}, \frac{5}{6}\right]$

B. $\left(\frac{1}{2}, \frac{5}{6}\right]$

C. $\left[\frac{1}{2}, \frac{5}{6}\right)$

D. $\left(\frac{1}{2}, \frac{5}{6}\right)$

Answer: A

Solution:

We have $|3x-2| \leq \frac{1}{2} \therefore \frac{-1}{2} \leq (3x-2) \leq \frac{1}{2}$
$\therefore \frac{-1}{2} \leq 3x-2 \quad \text{and} \quad 3x-2 \leq \frac{1}{2} \therefore \quad \frac{3}{2} \leq 3x \quad \text{and}$
$3x \leq \frac{5}{2} \quad \frac{1}{2} \leq x \quad \text{and} \quad x \leq \frac{5}{6} \therefore x \in \left[\frac{1}{2}, \frac{5}{6}\right]$
